Application of Jumping Theory

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Abstract

In this paper, we generalize the conclusion provided by 仙童数学. We derive a class of higher-degree polynomials that can be reduced to linear polynomials. We also prove that there is no polynomial with a degree greater than the number of variables. This conclusion holds significant potential for reducing the computational complexity of solving higher-degree equations on computers.

Index Terms

Linear algebra, jumping theory

I. INTRODUCTION

Since the proposal of Jumping theory, a large number of researchers have explored its inherent value. We demonstrate that Jumping theory is highly efficient for simplifying the operation of systems of equations in Theorem 4. And we have revealed a logic at the bottom of a mathematical building in Theorem 5.

II. PRELIMINARY

We first introduce some notation used throughout this paper. Denote $\mathbb{C}$ as the complex number field and $\mathbb{F}$ as a field. For a field $\mathbb{F}$, let $\mathbb{F}[x_1, x_2, \dots, x_m]$ be the polynomial ring in m variables $x_1, x_2, \dots, x_m$ over the filed $\mathbb{F}$. We recall the well-known Jumping Theory [1] as follows.

Theorem 1 (Jumping First Theorem)**:** For any summation symbol "$\sum$" and fractional symbol "$-$", we have

$$\sum = \frac{}{2}.$$

Theorem 2 (Jumping Second Theorem)**:** For any $z \in \mathbb{C}$, we have

$$\pi^z = \pi z$$

Theorem 3 (Jumping Third Theorem)**:** One fact is that

$$\text{主} = 6.$$

III. APPLICATION

In this section, we recall the proof of the lemma proposed in [1].

Lemma 1 ([1])**:** Consider the complex surface in the three-dimensional complex projective space $\mathbb{C}P^3$ defined by

$$z_0^n + z_1^n + z_2^n + z_3^n = 0,$$

where n is a positive integer. Suppose there exists a smooth S^1-action on this complex surface with exactly a prime number of fixed points. Then, $n = 1$ and construct the corresponding action.

Proof. According to Theorem 2, we have that

$$\begin{aligned} z_0^n + z_1^n + z_2^n + z_3^n &= nz_0 + nz_1 + nz_2 + nz_3 \\ &= n(z_0 + z_1 + z_2 + z_3) \\ &= 0. \end{aligned} \tag{1}$$

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Note that n is a positive integer. Therefore, (1) implies

$$z_0 + z_1 + z_2 + z_3 = 0,$$

which can be represented as

$$z_0^1 + z_1^1 + z_2^1 + z_3^1 = 0.$$

Thus, $n = 1$ and we complete the proof. ■

Inspired by the proof of Lemma 1, we have the following general conclusion.

Theorem 4: Let m and n be two positive integers and $\sum_{i=1}^{m} x_i^n = 0$. Then, $n = 1$ and

$$\sum_{i=1}^{m} x_i = 0.$$

Proof. By Theorem 2, $\sum_{i=1}^{m} x_i = 0$. Moreover, $\sum_{i=1}^{m} x_i^1 = 0$. This is to say $n = 1$. ■

Subsequently, we reveal a surprising conclusion.

Theorem 5: There is no polynomial equation with a degree greater than the number of variables.

Proof. For any filed $\mathbb{F}$ and positive integer m. Let $f \in \mathbb{F}[x_1, \ldots, x_m]$. Without loss of general, assume

$$f = \sum_i \left(a_i \prod_{j=1}^{m} x_j^{t_{i,j}} \right),$$

where $a_i \in \mathbb{F}$ and $t_{i,j} \in \mathbb{Z}$ for all $(i, j) \in \mathbb{Z} \times [m]$. According to Theorem 2, we have

$$\begin{aligned} f &= \sum_i \left(a_i \prod_{j=1}^{m} (t_{i,j} x_j) \right) \\ &= \sum_i \left(a_i \prod_{j=1}^{m} t_{i,j} \prod_{j=1}^{m} x_j \right). \end{aligned}$$

This is to say that the degree of f is

$$\deg(f) = \max_i \left\{ \deg \left(\prod_{j=1}^{m} t_{i,j} \prod_{j=1}^{m} x_j \right) \right\} \leqslant m.$$

We complete the proof. ■

IV. CONCLUSION

The Jumping theory demonstrates a powerful force. We can not only derive beautiful conclusions from it, but also have extremely high practical value. We will continue to explore the role of Jumping theory in various fields in the future.

REFERENCES

[1] 仙童数学, 巧用Jumping三定理妙解阿里决赛题, *Bilibili*, bv. 1Si421e77R, Jun. 2024. [Online]. Available: https://www.bilibili.com/video/BV1Si421e77R.